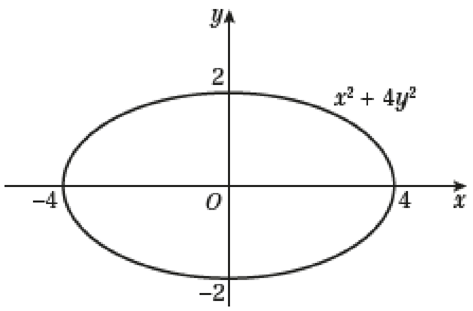
Exercise 2A

1 i a $x^2 + 4y^2 = 16 \Rightarrow \dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

So $a = 4$ and $b = 2$

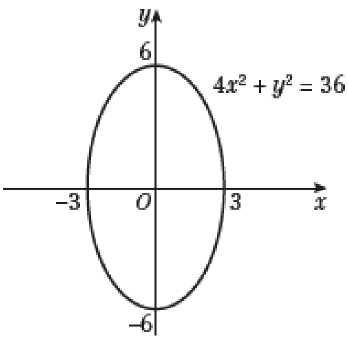


b Parametric equations

$x = 4\cos\theta, y = 2\sin\theta$

ii a $4x^2 + y^2 = 36 \Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$

So $a = 3$ and $b = 6$

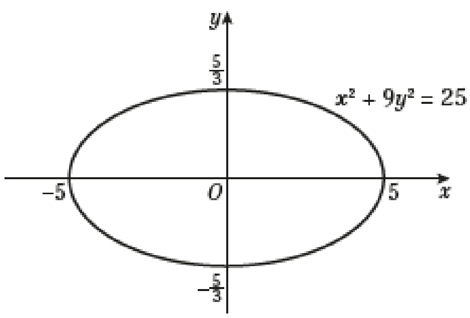


b Parametric equations

$x = 3\cos\theta, y = 6\sin\theta$

iii a $x^2 + 9y^2 = 25 \Rightarrow \dfrac{x^2}{25} + \dfrac{y^2}{\left(\frac{5}{3}\right)^2} = 1$

So $a = 5$ and $b = \frac{5}{3}$

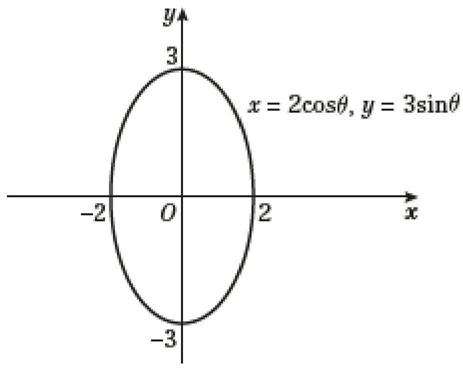


b Parametric equations

$x = 5\cos\theta, y = \frac{5}{3}\sin\theta$

2 i a $x = 2\cos\theta, y = 3\sin\theta$

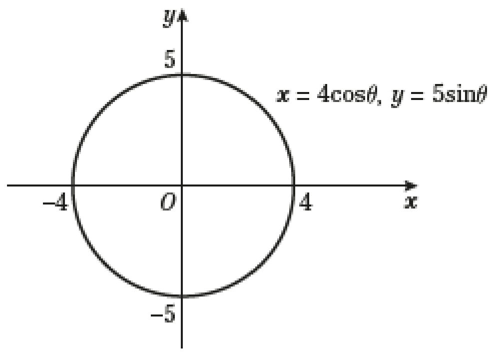
$-2 \leqslant x \leqslant 2; -3 \leqslant y \leqslant 3$



b $a = 2$ and $b = 3$, so the Cartesian equation is $\dfrac{x^2}{2^2} + \dfrac{y^2}{3^2} = 1$

ii a $x = 4\cos\theta, y = 5\sin\theta$

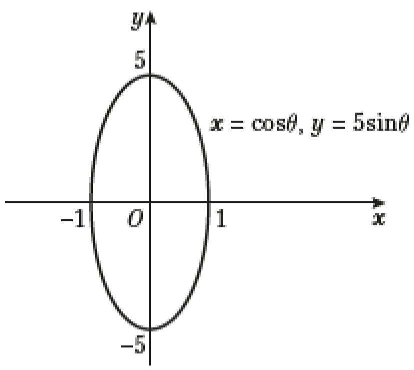
$-4 \leqslant x \leqslant 4; -5 \leqslant y \leqslant 5$



b $a = 4$ and $b = 5$, so the Cartesian equation is $\dfrac{x^2}{4^2} + \dfrac{y^2}{5^2} = 1$

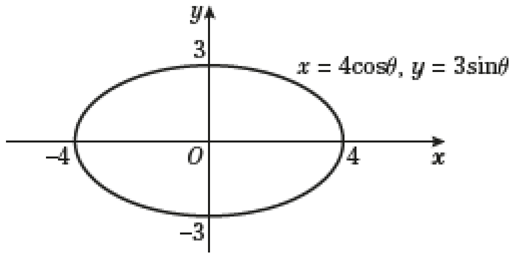
iii a $x = \cos\theta, y = 5\sin\theta$

$-1 \leqslant x \leqslant 1; -5 \leqslant y \leqslant 5$



2 iii b $a = 1$ and $b = 5$, so the Cartesian equation is $x^2 + \frac{y^2}{5^2} = 1$

iv a $x = 4\cos\theta, y = 3\sin\theta$
$-4 \leqslant x \leqslant 4; -3 \leqslant y \leqslant 3$

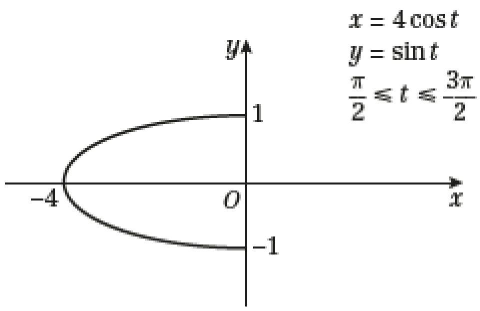


b $a = 4$ and $b = 3$, so the Cartesian equation is $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$

3 a Clearly P has coordinates $(a\cos\theta, a\sin\theta)$ and Q has coordinates $(b\cos\theta, b\sin\theta)$. Then by definition $R = (b\cos\theta, a\sin\theta)$.

b Since $\cos^2\theta + \sin^2\theta = 1$, we have that the locus described by R as θ varies from 0 to 2π is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
This is the equation of an ellipse.

c



Challenge

The matrix of a rotation of 45° anticlockwise is

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

If we apply this to the general vector $\begin{pmatrix} a\cos\theta \\ b\cos\theta \end{pmatrix}$

we get $\begin{pmatrix} \frac{a}{\sqrt{2}}\cos\theta - \frac{b}{\sqrt{2}}\sin\theta \\ \frac{a}{\sqrt{2}}\cos\theta + \frac{b}{\sqrt{2}}\sin\theta \end{pmatrix}$

Then we can compute:

$$\frac{(x+y)^2}{2a^2} + \frac{(x-y)^2}{2b^2} = \frac{\left(a\sqrt{2}\cos\theta\right)^2}{2a^2} + \frac{\left(-b\sqrt{2}\sin\theta\right)^2}{2b^2}$$

$$= \frac{2a^2\cos^2\theta}{2a^2} + \frac{2b^2\sin^2\theta}{2b^2}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$